NAG Toolbox for MATLAB

f08qu

1 Purpose

f08qu reorders the Schur factorization of a complex general matrix so that a selected cluster of eigenvalues appears in the leading elements on the diagonal of the Schur form. The function also optionally computes the reciprocal condition numbers of the cluster of eigenvalues and/or the invariant subspace.

2 Syntax

```
[t, q, w, m, s, sep, info] = f08qu(job, compq, select, t, q, 'n', n)
```

3 Description

f08qu reorders the Schur factorization of a complex general matrix $A = QTQ^{\mathrm{H}}$, so that a selected cluster of eigenvalues appears in the leading diagonal elements of the Schur form.

The reordered Schur form $\tilde{T}$ is computed by a unitary similarity transformation: $\tilde{T} = Z^{\mathrm{H}}TZ$. Optionally the updated matrix $\tilde{Q}$ of Schur vectors is computed as $\tilde{Q} = QZ$, giving $A = \tilde{Q}\tilde{T}\tilde{Q}^{\mathrm{H}}$.

Let $\tilde{T} = \begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix}$, where the selected eigenvalues are precisely the eigenvalues of the leading m by m sub-matrix T_{11}. Let $\tilde{Q}$ be correspondingly partitioned as $\begin{pmatrix} Q_1 & Q_2 \end{pmatrix}$ where Q_1 consists of the first m columns of Q. Then $AQ_1 = Q_1T_{11}$, and so the m columns of Q_1 form an orthonormal basis for the invariant subspace corresponding to the selected cluster of eigenvalues.

Optionally the function also computes estimates of the reciprocal condition numbers of the average of the cluster of eigenvalues and of the invariant subspace.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **job – string**

Indicates whether condition numbers are required for the cluster of eigenvalues and/or the invariant subspace.

job = 'N'

No condition numbers are required.

job = 'E'

Only the condition number for the cluster of eigenvalues is computed.

job = 'V'

Only the condition number for the invariant subspace is computed.

job = 'B'

Condition numbers for both the cluster of eigenvalues and the invariant subspace are computed.

Constraint: **job** = 'N', 'E', 'V' or 'B'.

2: **compq – string**

Indicates whether the matrix Q of Schur vectors is to be updated.

compq = 'V'

The matrix Q of Schur vectors is updated.

compq = 'N'

No Schur vectors are updated.

Constraint: **compq** = 'V' or 'N'.

3: **select**$(*)$ **– logical array**

Note: the dimension of the array **select** must be at least $\max(1, \mathbf{n})$.

Specifies the eigenvalues in the selected cluster. To select a complex eigenvalue λ_j, **select**(j) must be set **true**.

4: **t**(**ldt**,$*$) **– complex array**

The first dimension of the array **t** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n upper triangular matrix T, as returned by f08ps.

5: **q**(**ldq**,$*$) **– complex array**

The first dimension, **ldq**, of the array **q** must satisfy

if **compq** = 'V', **ldq** $\geq \max(1, \mathbf{n})$;
if **compq** = 'N', **ldq** ≥ 1.

The second dimension of the array must be at least $\max(1, \mathbf{n})$ if **compq** = 'V' and at least 1 if **compq** = 'N'

If **compq** = 'V', **q** must contain the n by n unitary matrix Q of Schur vectors, as returned by f08ps.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **select** The second dimension of the array **t**.

n, the order of the matrix T.

Constraint: **n** ≥ 0.

5.3 Input Parameters Omitted from the MATLAB Interface

ldt, ldq, work, lwork

5.4 Output Parameters

1: **t**(**ldt**,∗) – **complex array**

The first dimension of the array **t** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

t contains the updated matrix $\tilde{T}$.

2: **q**(**ldq**,∗) – **complex array**

The first dimension, **ldq**, of the array **q** must satisfy

if **compq** = 'V', **ldq** $\geq \max(1, \mathbf{n})$;
if **compq** = 'N', **ldq** ≥ 1.

The second dimension of the array must be at least $\max(1, \mathbf{n})$ if **compq** = 'V' and at least 1 if **compq** = 'N'

If **compq** = 'V', **q** contains the updated matrix of Schur vectors; the first m columns of **q** form an orthonormal basis for the specified invariant subspace.

If **compq** = 'N', **q** is not referenced.

3: **w**(∗) – **complex array**

Note: the dimension of the array **w** must be at least $\max(1, \mathbf{n})$.

The reordered eigenvalues of $\tilde{T}$. The eigenvalues are stored in the same order as on the diagonal of $\tilde{T}$.

4: **m** – **int32 scalar**

m, the dimension of the specified invariant subspace, which is the same as the number of selected eigenvalues (see **select**); $0 \leq m \leq n$.

5: **s** – **double scalar**

If **job** = 'E' or 'B', **s** is a lower bound on the reciprocal condition number of the average of the selected cluster of eigenvalues. If **m** = 0 or **n**, **s** = 1.

If **job** = 'N' or 'V', **s** is not referenced.

6: **sep** – **double scalar**

If **job** = 'V' or 'B', **sep** is the estimated reciprocal condition number of the specified invariant subspace. If **m** = 0 or **n**, **sep** = $\|T\|$.

If **job** = 'N' or 'E', **sep** is not referenced.

7: **info** – **int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **job**, 2: **compq**, 3: **select**, 4: **n**, 5: **t**, 6: **ldt**, 7: **q**, 8: **ldq**, 9: **w**, 10: **m**, 11: **s**, 12: **sep**, 13: **work**, 14: **lwork**, 15: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed matrix $\tilde{T}$ is similar to a matrix $(T+E)$, where

$$\|E\|_2 = O(\epsilon)\|T\|_2,$$

and ϵ is the ***machine precision***.

s cannot underestimate the true reciprocal condition number by more than a factor of $\sqrt{\min(m, n-m)}$. **sep** may differ from the true value by $\sqrt{m(n-m)}$. The angle between the computed invariant subspace and the true subspace is $\dfrac{O(\epsilon)\|A\|_2}{\text{sep}}$.

The values of the eigenvalues are never changed by the reordering.

8 Further Comments

The real analogue of this function is f08qg.

9 Example

```
job = 'Both';
compq = 'Vectors';
select = [true;
     false;
     false;
     true];
t = [complex(-6.0004, -6.9999), complex(0.3637, -0.3656), complex(-0.188,
...
     +0.4787), complex(0.8784999999999999, -0.2539);
        complex(0, +0), complex(-5, +2.006), complex(-0.0307, -0.7217),
complex(-0.229, +0.1313);
           complex(0, +0), complex(0, +0), complex(7.9982, -0.9964),
complex(0.9357, +0.5359);
       complex(0, +0), complex(0, +0), complex(0, +0), complex(3.0023, -
3.9998)];
q = [complex(-0.8347, -0.1364), complex(-0.06279999999999999, +0.3806),
...
     complex(0.2765, -0.08459999999999999), complex(0.0633, -0.2199);
      complex(0.0664, -0.2968), complex(0.2365, +0.524), complex(-0.5877,
-0.4208), complex(0.0835, +0.2183);
     complex(-0.0362, -0.3215), complex(0.3143, -0.5473), complex(0.0576,
-0.5736), complex(0.0057, -0.4058);
      complex(0.0086, +0.2958), complex(-0.3416, -0.0757), complex(-0.19,
-0.16), complex(0.8327, -0.1868)];
[tOut, qOut, w, m, s, sep, info] = f08qu(job, compq, select, t, q)
```

```
tOut =
   -6.0004 - 6.9999i   -0.9370 + 0.1096i   -0.3094 + 0.4442i   -0.0151 +
0.4283i
         0               3.0023 - 3.9998i    0.2223 - 0.2556i    1.0237 +
0.3045i
         0                     0             -5.0000 + 2.0060i   -0.0270 +
0.7057i
         0                     0                   0              7.9982 -
0.9964i
```

```
qOut =
   -0.8347 - 0.1364i   -0.0600 + 0.2044i   -0.0189 - 0.3783i    0.2816 -
0.1289i
    0.0664 - 0.2968i   -0.1109 - 0.3071i   -0.3471 - 0.4724i   -0.5713 -
0.3618i
   -0.0362 - 0.3215i   -0.0545 + 0.2685i   -0.1926 + 0.6106i    0.0457 -
0.6389i
    0.0086 + 0.2958i   -0.8779 - 0.0480i    0.3207 + 0.0071i   -0.0387 -
0.1873i
w =
  -6.0004 - 6.9999i
   3.0023 - 3.9998i
  -5.0000 + 2.0060i
   7.9982 - 0.9964i
m =
           2
s =
    0.9807
sep =
    5.4892
info =
           0
```